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CAM-CLAY PREDICTIONS OF UNDRAINED STRENGTH

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INTRODUCTION

It is important to be able to predict the strength of overconsolidated and normally-consolidated clays because most natural soil deposits are lightly to heavily overconsolidated or at least have an overconsolidated crustal layer. Laboratory and field testing are essential to the thorough investigation of a site. However, the information obtained is limited in many ways because of problems associated with sampling, testing, and costs. It is therefore advantageous to represent the undrained shear strength in a simple manner, using as few soil constants as possible. The intent of this paper is to review documented and published data found in the geotechnical literature and compare observed soil behavior under undrained conditions with predictions made using the critical-state Cam-Clay concepts presented by Schofield and Wroth (66). It is shown that the theory is an effective stress approach that encompasses total stress analyses such as that presented in the Stress History and Normalized Soil Engineering Properties SHANSEP method by Ladd and Foott (40).

The critical-state Cam-Clay concept of soil mechanics is a plasticity theory for soil behavior based on an interdependent relationship between strength, effective stress, and water content. Schofield and Wroth (66) review the theory in detail and amendments to the model have been proposed by Roscoe and Burland (63), Egan (21), Pender (58), Van Eekelen and Potts (84) and others. The model is simple in that only two soil constants are required to represent the undrained shear strength of a soil for any degree of overconsolidation (OCR). These soil parameters are: (1) the effective friction angle (ϕ'); and (2) the critical-state pore pressure parameter (Λ_0), that may be obtained experimentally from the results of one or more consolidated undrained triaxial compression tests (CIU or CK_0U tests).

This study includes a summary of the static undrained strength data of clay

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TABLE 1.—Soil

Number (1)	Soil (2)	LL (3)	PI (4)	ϕ' (5)	C_c (6)	C_s (7)
1	Weald	43	25	22.0	0.214	0.081
2	Kaolin	72	31	23.5	0.573	0.170
				22.0	0.580	0.165
3	San Francisco Bay Mud	88	45	35.2	0.665	0.221
4	London Clay	78	52	18.4	0.371	0.143
5	Keuper Marl	32	14	25.9	0.272	0.058
6	Portland	55	26	32.0	N/A	N/A
7	Illite	57	31	24.6	0.431	0.138
8	Boston Blue	41	21	26.8	0.338	0.138
9	Bangkok	65	41	25.4	0.511	0.092
10	Oslo	39	18	27.0	0.175	0.035
11	Bradwell	95	65	20.0	0.714	0.152
12	Bentler	63	33	22.7	N/A	N/A
13	Shellhaven	110	80	23.0	0.937	0.051
14	Calcium Montmorillonite	203	169	12.5	0.979	0.440
15	Newfield	28	10	30.5	0.210	0.018
16	Vienna	47	25	25.8	0.299	0.055
17	Virginia Coastal	54	27	28.5	0.444	0.055
18	Spestone Kaolin	72	32	22.6	0.693	0.069
19	Japanese	64	37	33.7	0.403	0.078
20	Little Belt	127	91	21.0	0.889	0.442
21	Atchafalaya	95	75	21.0	N/A	N/A
22	Soft Bangkok	85	49	20.0	N/A	N/A
23	Milazzo	61	33	23.0	0.431	0.111
24	Calcium Illite	85	48	24.2	0.758	0.274
25	Mexico Volcanic	426	286	47.0	7.145	2.577
26	Drammen	33	15	28.0	0.265	0.055
27	Backswamp	70	40	22.2	0.583	0.163
28	Kars Leda	45	21	28.3	1.121	0.021
29	Portsmouth	35	15	21.0	0.299	0.014
30	Willard Bay	N/A	N/A	37.0	0.933	N/A
31	Plastic Holocene	65	38	32.9	0.640	0.087
32	Ghana	42	29	20.8	0.182	0.058
33	Halloysite	62	26	34.3	0.304	0.053
34	Simple Clay	N/A	N/A	23.1	0.207	0.083
35	Seattle	52	26	28.8	0.260	0.081
36	Rang de Fleuve	73	46	28.6	N/A	N/A
37	Sodium Illite	79	44	20.7	0.362	0.200
38	Grundite	55	29	32.3	0.359	0.177
39	Terra Roxa	43	22	29.2	0.216	0.046
40	Amuay	71	42	29.9	0.488	N/A
41	Scott	34	12	33.4	0.138	0.037
42	Connecticut Varved	65	39	20.9	0.569	0.101
		35	12			
43	Toledo	42	23	20.0	0.288	0.067
44	Kawasaki	65	31	35.9	0.668	0.058
45	Tjipanundjang	165	46	35.0	N/A	N/A

Properties

S_u/σ'_{vo} (n.c.) (8)	Λ_0 (9)	r (10)	Type test (11)	Reference (12)
0.279	0.531	0.994	CIU	Henkel 1960
(0.313)[a]	0.389	0.984	CIU	Amerasinghe and Parry, 1975
(0.308)[a]	0.319	0.985	CKoU	
0.432	0.568	0.983	CIU	Mitchell, 1976
0.250	0.384	0.970	CIU	Henkel, 1960
(0.256)[a]	0.758	0.999	CIU	Brown et al., 1975
0.300	0.767	0.999	CKoU	Ladd and Foott, 1974
0.341	0.491	0.993	CIU	France and Sangrey, 1977
0.200	0.752	0.998	CKoU	Ladd and Foott, 1974
0.270	0.727	0.998	CKoU	Ladd and Foott, 1974
0.380	0.618	0.988	CIU	Simons, 1960b
(0.300)[a]	0.376	0.959	CIU	Skempton, 1961
0.310	0.590	0.999	CIU	Togrol, 1965
0.205	0.779	0.994	FV, CIU	Skempton and Henkel, 1953
0.221	0.193	0.919	CIU	Mesri and Olson, 1970
0.475	0.505	0.999	CIU	Sangrey, Henkel and Esrig, 1969
0.356	0.671	0.997	Q-DS	Hvorslev, 1960
0.409	0.482	0.955	CIU	Swanson and Brown, 1977
0.210	0.704	0.992	CIU	Parry and Nadarajah, 1973
0.400	0.727	0.999	CIU	Shibata and Karube, 1969
0.320	0.454	0.967	Q-DS	Hvorslev, 1960
0.240	0.772	0.998	CKoU	Ladd and Foott, 1974
0.255	0.653	0.992	CIU	Moh, Nelson, and Brand, 1969
0.333	0.630	0.995	CIU	Croce et al., 1969
0.250	0.590	0.999	CIU	Olson, 1962
0.430	0.788	0.997	CKoU	Lo, 1962
0.310	0.875	0.972	CIU	Simons, 1960a
0.280	0.641	0.998	CIU	Whitman, 1960
0.265	0.998	0.950	UU, CIU	Raymond, 1972
0.230	0.709	0.994	CKoU	Simon, Christian, and Ladd, 1974
0.360	0.671	0.982	CIU	Gibbs et al., 1960
0.335	0.714	0.999	CIU	Koutsoftas and Fischer, 1976
0.380	0.290	0.974	CIU	deGraft-Johnson et al., 1969
0.418	0.835	0.992	CIU	Taylor and Bacchus, 1969
0.290	0.550	0.998	CIU	Ladd, 1964
0.372	0.515	0.997	CIU	Sheriff, Wu and Bostrum, 1972
0.378	0.920	0.995	CIU	Tavenas et al., 1978
0.340	0.369	0.954	CIU	Olson and Hardin, 1963
0.356	0.413	0.997	CIU	Perloff and Osterberg, 1963
0.313	0.872	0.978	CIU	da Cruz, 1963
0.342	0.596	0.993	CIU	Lambe, 1963
0.231	0.922	0.999	CIU	Ladanyi et al., 1965
0.163	0.724	0.998	CKoU	Ladd and Foott, 1974
0.200	0.619	0.999	CIU	Wu, Chang, and Ali, 1978
0.370	0.842	0.994	CIU, UU	Ladd and Lambe, 1963
N/A	0.469	0.999	CIU	Wesley, 1974

TABLE 1.—

(1)	(2)	(3)	(4)	(5)	(6)	(7)
46	Weirton	51	25	19.0	N/A	N/A
47	Concord Blue	32	10	24.8	0.163	0.039
48	Agnew	54	24	25.0	0.159	0.048
49	Lagunillas	61	37	26.5	0.771	0.127
50	Drammen	N/A	N/A	30.7	0.599	0.127
51	Liskeard	56	33	26.1	N/A	N/A
52	Vicksburg	63	39	25.9	N/A	N/A
53	Massachusetts	21	6	30.5	N/A	N/A
54	Moose River Muskeg	N/A	N/A	47.7	N/A	N/A
55	Ottawa Estuarine	53	26	35.3	N/A	N/A
56	Lilla Edet	61	32	24.3	N/A	N/A
57	New York Varved	50	25	N/A	N/A	N/A
58	Khor-Al-Zubair	55	35	27.3	N/A	N/A
59	Texcoco	343	279	34.0	N/A	N/A
60	Hokkaido Silt A	52	21	37.2	0.299	0.058
				35.1	0.290	0.060
61	Hokkaido Silt B	51	21	35.1	0.193	0.030
				34.9	N/A	N/A
62	Hokkaido Clay	72	32	36.1	0.412	0.058
				34.0	0.410	0.060
63	Fao	39	20	36.9	N/A	N/A
64	Saint Alban	45	22	27.0	N/A	N/A
65	Kanpur Clay	38	18	29.0	0.284	0.060
66	Rann of Kutch	91	49	26.0	0.610	0.340
67	Spestone Kaolin	76	37	17.2	N/A	N/A
68	Kaolinite	57	25	29.2	N/A	N/A
69	New England	N/A	20	32.0	N/A	N/A
70	Ohio Silt	24	4	32.9	0.133	0.040
71	Drammen Clay	55	26	25.1	N/A	N/A
72	Länsisalmi	78	46	19.5	1.059	0.401
73	Sault Ste Marie	55	32	28.9	0.165	0.050
74	Bath Kaolinite	48	15	24.5	0.189	0.045
75	Mödndal	55	25	27.0	N/A	N/A
76	Bangalore Montmorillonite	580	495	12.5	N/A	N/A
77	Bangalore Kaolinite	49	20	25.5	N/A	N/A
78	Kinnegar	88	58	27.0	N/A	N/A
79	Calcutta	58	20	30.2	N/A	N/A
80	Regina	83	54	20.0	N/A	N/A
81	Long Island Coastal	64	34	22.8	0.222	0.068
82	Hackensack Varved	65	35	19.0	0.481	0.068
		40	25			
83	New Providence	31	10	30.5	N/A	N/A
84	Alaskan Gulf	35	14	34.5	N/A	N/A
85	East Atchafalaya					
	$Kc = 1.00$	79	53	21.7	0.479	N/A
	$Kc = 0.67$			18.8		
	$Kc = 0.50$			21.5		
86	Buckshot Clay					
	$Kc = 1.00$	57	36	26.7	0.402	N/A

Continued

(8)	(9)	(10)	(11)	(12)
N/A	0.498	0.982	CIU	D'Appolonia et al., 1966
0.355	0.684	0.998	CIU	Egan, 1977
0.308	0.491	0.973	CKoU	Egan, 1977
0.305	0.589	0.992	CIU	Ladd and Lambe, 1963
0.285	0.781	0.996	CIU	Van Eekelen and Potts, 1978
0.298	0.847	0.990	CIU	Raymond, 1973
0.287	0.729	0.999	CIU	Ladd, 1962
0.416	0.223	0.999	CIU	Wissa, Ladd, Lambe, 1965
0.683	0.541	0.996	CIU	Adams, 1965
0.300	0.828	0.996	CIU	Kenney and Watson, 1961
0.290	0.372	0.997	CIU	Bjerrum and Simons, 1960
0.328	0.620	0.990	FV	Leathers and Ladd, 1978
0.360	0.684	0.999	CIU	Hanzawa, 1977a
0.420	0.725	0.941	CIU	Alberro and Santoyo, 1973
0.420	0.787	0.999	CIU	Mitachi and Kitago, 1976
0.400	0.765	0.997	CK$_o$U	
0.362	0.851	0.999	CIU	Mitachi and Kitago, 1976
0.361	0.795	0.998	CK$_o$U	
0.410	0.791	0.997	CIU	Mitachi and Kitago, 1976
0.360	0.786	0.998	CK$_o$U	
0.530	0.584	0.999	CIU,	Hanzawa, 1977b
0.290	0.972	0.994	CIU,	Tavenas et al., 1978b
0.295	0.662	0.999	CIU	Yudhbir and Varadarajah, 1974
0.326	0.454	0.995	CIU	Yudhbir and Varadarajah, 1974
0.184	0.740	0.999	CAU	Sketchley and Bransby, 1973
0.492	0.130	0.999	CIU	Broms and Casbarian, 1965
0.325	0.715	0.999	CIU	Ladd, 1976
N/A	0.709	0.993	CIU	Montgomery, 1978
0.218	0.881	0.999	CIU	Fischer et al., 1978
0.215	0.364	0.957	CIU	Korhonen, 1977
0.327	0.743	0.887	CIU	Wu, Douglas, and Goughnour, 1962
0.425	0.431	0.996	CIU	Broms and Ratnam, 1963
0.254	0.720	0.993	CIU	Karlsson and Pusch, 1967
0.211	0.166	0.999	CIU	Sridharan et al., 1971
0.422	0.326	0.999	CIU	Sridharan et al., 1971
0.338	0.857	0.944	CAU, CIU	Crooks and Graham, 1976
0.342	0.544	0.984	CAU	Gangopadhyay et al., 1974
0.308	0.556	0.999	CIU	Widger and Fredlund, 1979
0.271	0.482	0.951	CIU	Swanson and Brown, 1977
0.158	0.760	0.986	CIU	Saxena, Hedberg, and Ladd, 1978
N/A	0.720	0.998	CIU	Abeyesekera, Lovell, and Wood, 1979
0.408	0.698	0.999	CIU	Singh and Gardner, 1979
0.282	0.599	0.993	CIU	Donaghe and Townsend, 1978
0.258	0.516	0.997	CAU	
0.281	0.488	0.998	CAU	
0.335	0.684	0.992	CIU	Donaghe and Townsend, 1978

TABLE 1.—

(1)	(2)	(3)	(4)	(5)	(6)	(7)
	$Kc = 0.67$			22.1		
	$Kc = 0.50$			24.9		
87	Kyoto	88	57	32.5	N/A	N/A
88	Guanabara Bay	140	90	25.1	N/A	N/A
89	Kodiak Island	30	14	42.2	0.124	0.056
90	Winnipeg Clay	94	60	12.8	0.996	0.240
91	Sydney Kaolin	50	16	30.7	N/A	N/A
92	Weald Clay	46	22	25.9	0.193	N/A
				25.0	0.195	N/A
93	Kaolinite vertical	N/A	N/A	27.8	N/A	N/A
	horizontal			29.2		
94	Kaolin	N/A	N/A	24.7	N/A	N/A
95	Beaumont Clay	67	41	24.0	N/A	N/A
96	Drammen Clay	57	27	30.7	0.526	0.043
					0.506	0.042

[a] Extrapolated from S_u/σ'_{v0} (o.c.) data.

and silt soils that are normally-consolidated to lightly and heavily overconsolidated. The data are reported by various researchers from countries all over the world. Since the soils come from such a wide variety of sources, a comparison of predicted and experimental soil behavior should prove a true test to applicability of the critical-state theory.

Soil Properties

The various journals of soil mechanics and geotechnical symposia proceedings were reviewed in order to provide a data bank for this study. Data from a total of 96 different soils were compiled during this research effort. Relevant soil properties for each of these soils are summarized in Table 1. Included are the plasticity characteristics, consolidation parameters, and shear strength data, as reported by the researchers who tested these materials.

Index properties contained in Table 1 include liquid limit (LL) and plasticity index (PI). Fig. 1 shows that the soils classify as low to medium to high plasticity clays and silts. The virgin compression index (C_c) and swelling index (C_s) have been determined from the results of one-dimensional consolidation tests conducted on the clay and silt soils. In general, strength data was obtained from consolidated-undrained shear tests with pore pressure measurements. The peak undrained shear strength to overburden ratio (S_u/σ'_{v0}) and effective stress friction angle (ϕ') from the normally-consolidated range of each soil are given in Table 1. The overburden pressure (σ'_{v0}) represents the initial vertical consolidation stress applied to the specimens before undrained shear to failure. The critical-state pore pressure parameter (Λ_0) and sample correlation coefficient (r) have been determined from linear regression analyses. The critical-state parameter is the basis for subsequent sections of this paper.

A variety of shear tests were used by the experimenters in order to investigate undrained behavior. The majority of the soils (approx 75%) were subjected

Continued

(8)	(9)	(10)	(11)	(12)
0.305	0.730	0.996	CAU	
0.320	0.703	0.997	CAU	
0.530	0.211	0.999	CIU	Akai and Adachi, 1965
0.253	0.880	0.935	CIU, UU	Costa Filho, Werneck, and Collet, 1977
0.510	0.672	0.979	CIU	Sparrow, Swanson, and Brown, 1978
0.211	0.556	0.997	CIU	Crawford, 1964
0.410	0.704	0.972	CKoU	Poulos, 1978
0.323	0.683	0.999	CIU	Henkel and Sowa, 1964
0.256	0.696	0.998	CKoU	
N/A	0.736	0.991	CIU	Duncan and Seed, 1966
N/A	0.673	0.984	CIU	
0.238	0.673	0.990	CIU	Wroth and Loudon, 1967
0.267	0.624	0.988	CIU	Mahar and Ingram, 1979
0.280	0.815	0.999	CIU	Andersen et al., 1980
0.210	0.768	0.999	CKoU	

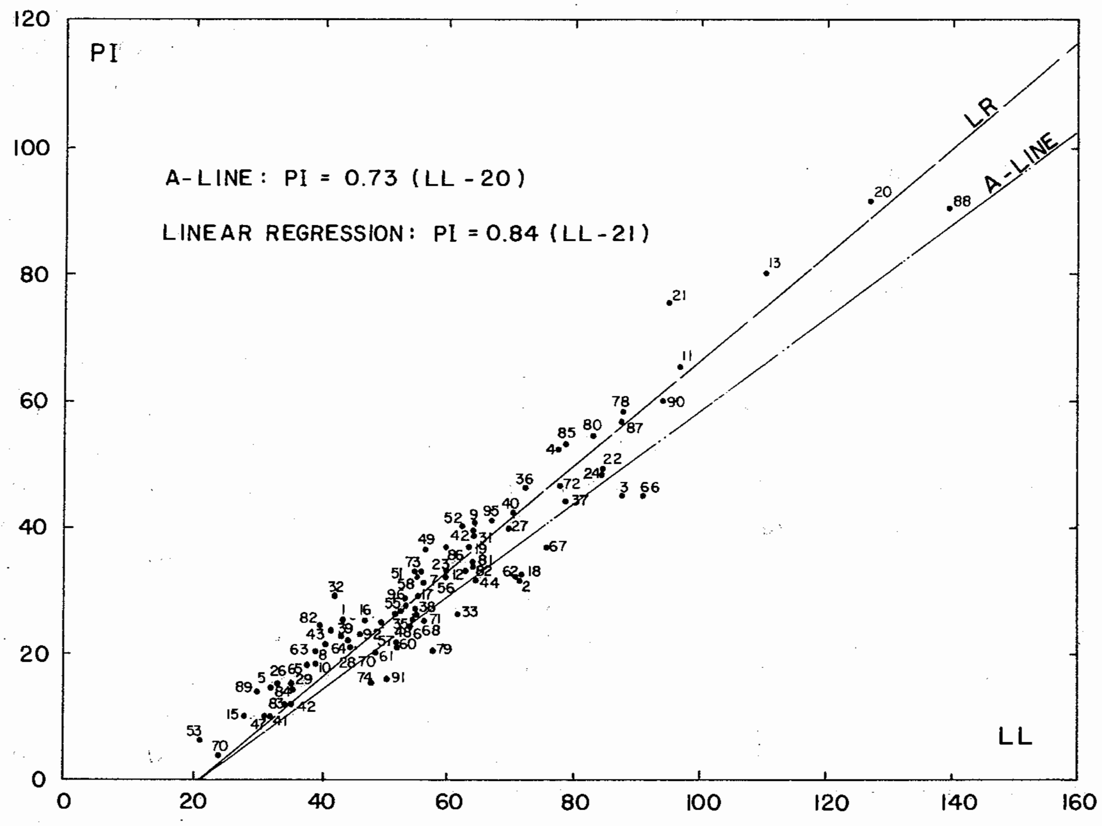

FIG. 1.—Plasticity Characteristics of Clay and Silt Soils

to isotropically-consolidated undrained triaxial shear tests (CIU) with pore pressure measurements. About 15% of the soils were tested in the triaxial apparatus using anisotropic-consolidation (CAU) or consolidation conditions of no lateral strain (CK_oU). A few soils were investigated using "quick" direct shear tests (Q-DS), unconsolidated-undrained tests (UU), and direct simple shear devices (CK_0U-DSS). One soil (No. 67) was tested in the plane-strain apparatus (PS-CKoU). Nine of the soils were subjected to both CIU and CKoU tests by their experimenters.

It should be noted, however, that many factors were not evaluated in this study. Some of these factors include: (1) Type of shear test (triaxial, simple shear, etc.); (2) soil structure (virgin, remolded, sedimented, etc.); (3) sensitivity; (4) differences in laboratory equipment and research personnel; (5) lateral anisotropy; (6) inhomogeneity; (7) rate of testing; and (8) activity or clay fraction, and other variables. Instead, it was hoped that the important basic trends in the behavior of clayey soils would be observed by considering data from many different sources.

Undrained Strength of Overconsolidated Clay

The critical-state concept can be used to predict the undrained strength of overconsolidated soils. For an isotropically-consolidated soil that has undergone a simple load-unload cycle in its stress history, it may be assumed that the stress path of an overconsolidated soil will reach the same failure point on the critical-state line (CSL) as a normally-consolidated sample at the same water content (see Fig. 2). This idea is similar to the equivalent pressure concept formulated by Hvorslev (30) and used by Togrol (83), Schofield and Wroth (66), and Pender (58). The application of this hypothesis to the model yields:

$$\frac{\dfrac{S_u}{\sigma'_{v0}}\text{ (overconsolidated)}}{\dfrac{S_u}{\sigma'_{v0}}\text{ (normally-consolidated)}} = \mathrm{OCR}^{\Lambda_0} \quad \text{. (1)}$$

in which the overconsolidation ratio is defined as OCR $= \sigma'_{v_{max}}/\sigma'_{v0}$; and Λ_0 is termed the critical-state pore pressure parameter. Theoretically, the parameter $\Lambda_0 = 1 - (C_{si}/C_{ci})$, in which C_{si} and C_{ci} = the isotropic swelling and compression indices, respectively. As a close approximation, Atkinson and Bransby (7) have suggested that the parameter $\Lambda_0 = 1 - (C_s/C_c)$, in which C_s and C_c = the conventional parameters obtained from consolidation tests. Values of Λ_0 for clays and silts should theoretically lie within the range $0 \leq \Lambda_0 \leq 1$.

Mitachi and Kitago (51) have further shown this relationship to be applicable for anisotropically-consolidated soils. Atkinson and Bransby (7) have derived a similar expression that includes the effect of K_0, the coefficient of earth pressure at rest. For simplicity, it is assumed that Eq. 1 applies for both isotropic and anisotropic conditions and for peak and critical state strengths.

It is well recognized that an overconsolidated soil exhibits a higher shear strength than a normally-consolidated sample of the material at the same confining stress level. Ladd and Foott (40) have incorporated this effect in their SHANSEP method by using normalized plots of shear strength-overburden ratios (S_u/σ'_{v0}) and overconsolidation ratio (OCR). Insight into the actual behavior of overconsolidation clays is obtained through the normalization of S_u/σ'_{v0} (overconsolidated) values to S_u/σ'_{v0} (normally-consolidated). For comparison, data from many different clays that have been published in the geotechnical literature have been compiled for this study. As shown by Fig. 3, the undrained shear strength to overburden ratio may be represented by a power function of the overconsolidation ratio, in a manner similar to that expressed by Eq. 1. Ladd et al. (41) have presented a similar format for the SHANSEP approach. Numerals shown in Fig. 3 refer to clay and silt soils listed in Table 1.

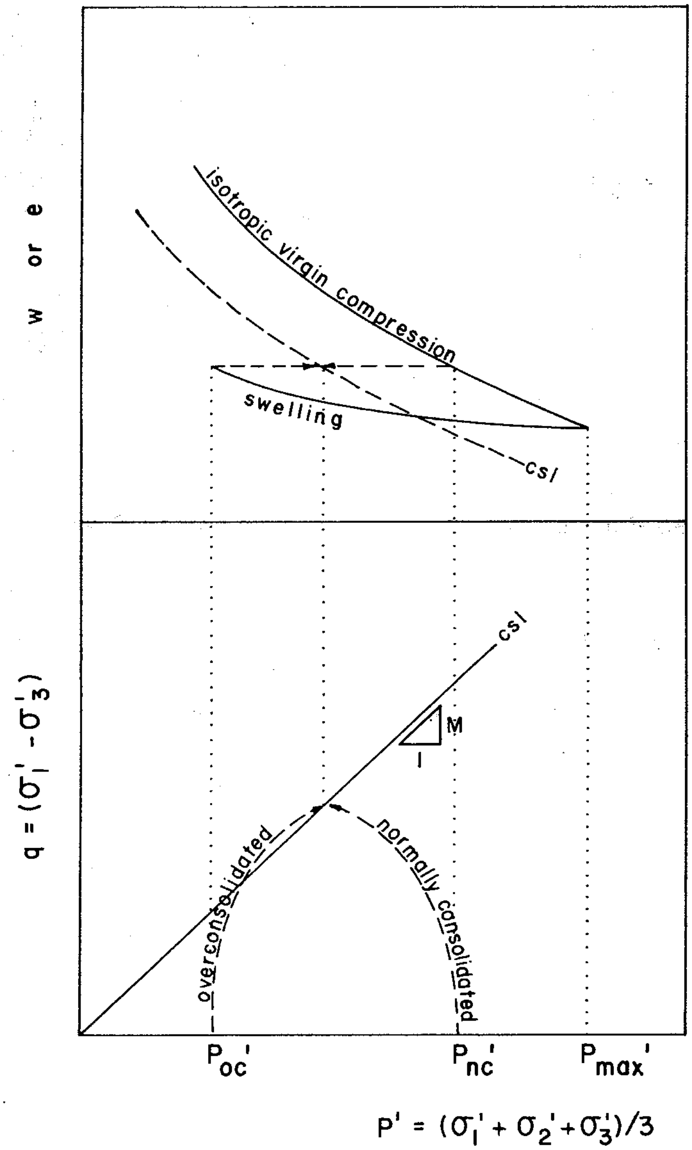

FIG. 2.—Equivalent Pressure Concept for Isotropically Normally-Consolidated and Overconsolidated Soil

The parameter Λ_0 may be experimentally determined from a knowledge of strengths at various levels of OCR. In this study, the critical-state pore pressure parameter (Λ_0) is defined by:

$$\Lambda_0 = \frac{\log\left[\dfrac{S_u}{\sigma'_{v0}}(\text{o.c.})\right] - \log\left[\dfrac{S_u}{\sigma'_{v0}}(\text{n.c.})\right]}{\log\,[\text{OCR}]} \quad (2)$$

or more simply as the slope of the linear relationship between S_u/σ'_{v0} (overconsolidated) and OCR on a log-log plot.

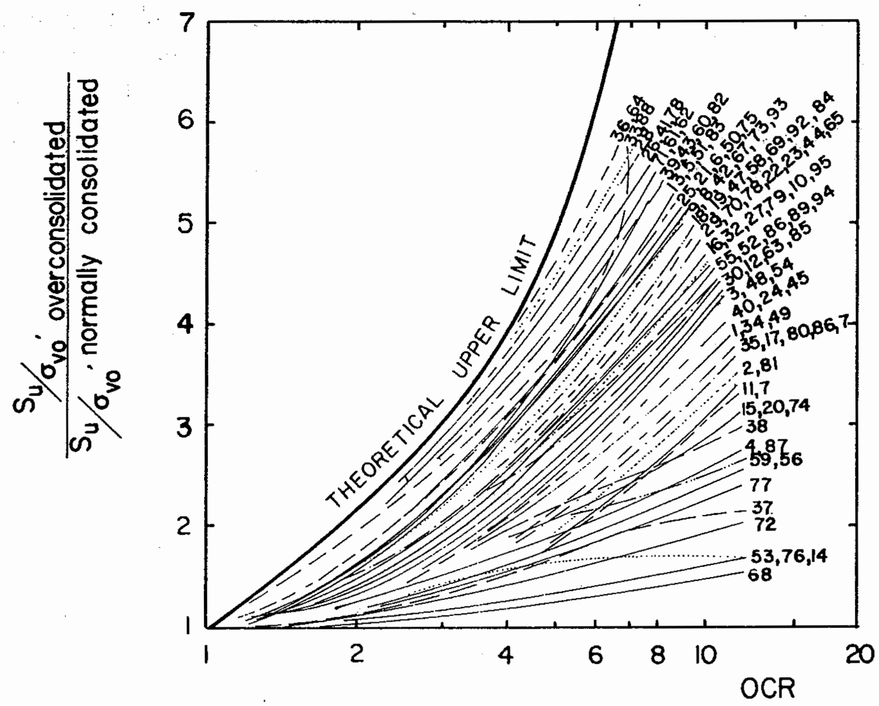

FIG. 3.—Observed Relationships Between S_u/σ'_{v0} and OCR for 96 Different Soils

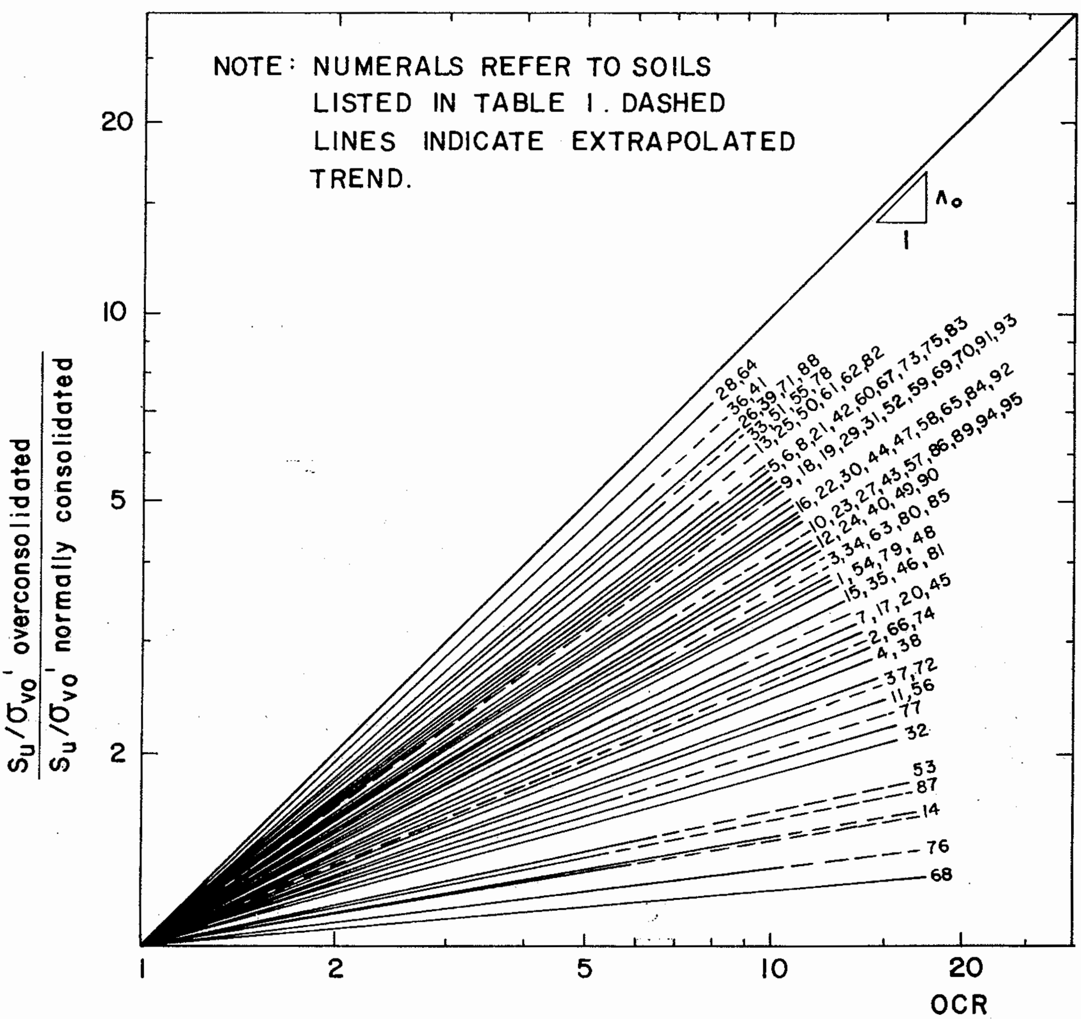

FIG. 4.—Normalized Experimental Relationships Between S_u/σ'_{v0} and OCR

Linear regression analyses have been conducted between log $[S_u/\sigma'_{v0}]$ and log [OCR] to determine the value of Λ_0 for each soil. Experimentally determined values of Λ_0 are listed in Table 1 for the clays and silts included in this study. Sample correlation coefficients are generally high ($r > 0.98$), indicating that Λ_0 is essentially constant with OCR. The trend of S_u/σ'_{v0} with OCR is clarified through Fig. 4. Values are seen to range from 0–1, as predicted by the critical-state theory. In fact, the variation of Λ_0 may be represented by a normal distribution with mean of 0.64 and standard deviation of ± 0.18. The higher observed values of Λ_0 appear to be associated with highly sensitive natural clays.

It is important to note that the critical-state pore pressure parameter has been determined from a total stress approach, similar to that used in the SHANSEP method of Ladd and Foott (40). In the preceding section, no direct pore pressure measurements have been used to determine the value of Λ_0.

Undrained Strength of Normally-Consolidated Clay

The Cam-Clay theory presented by Schofield and Wroth (66) follows the basic physical laws governing the conservation of energy. The concepts of work and stored energy are applied using principles of recoverable (elastic) and irrecoverable (plastic) strains. For initially isotropic conditions, it is derived that the undrained shear strength to overburden ratio for normally-consolidated

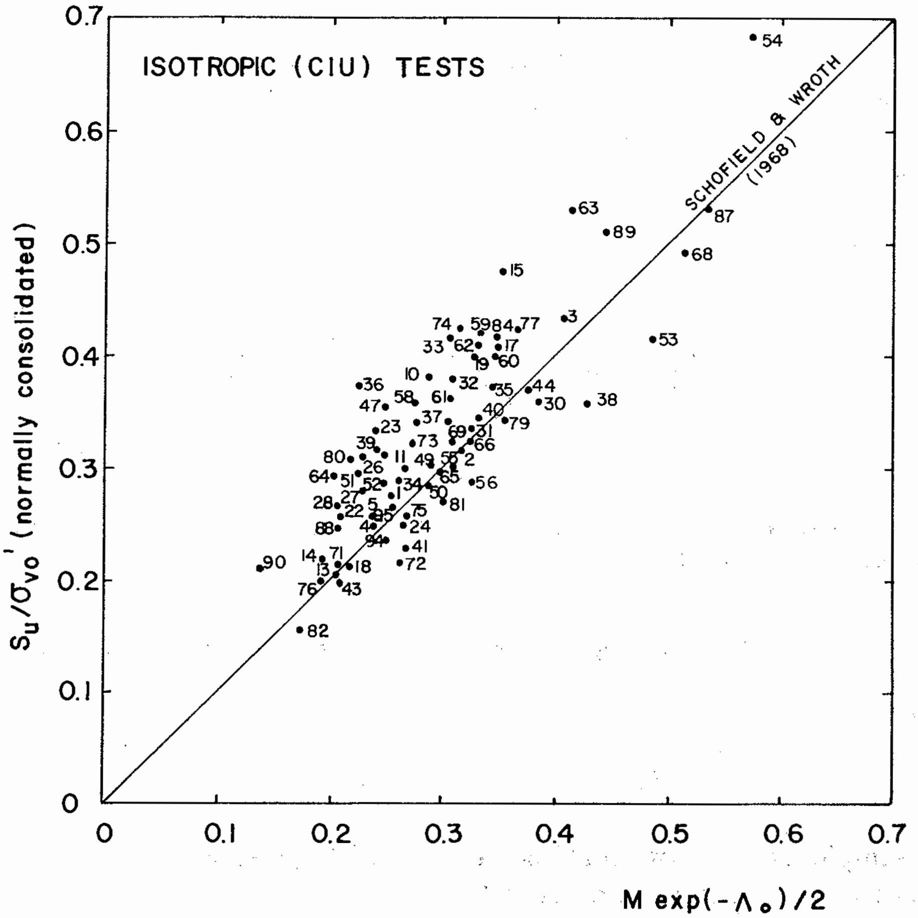

FIG. 5.—Comparison Between Measured and Predicted S_u/σ'_{v0} for Isotropically Normally-Consolidated Clays

soils is represented in terms of effective stresses by:

$$\frac{S_u}{\sigma'_{v0}} \text{(n.c.)} = \frac{3 \sin \phi' \exp(-\Lambda_0)}{(3 - \sin \phi')} \quad \ldots \quad (3)$$

in which ϕ' = the effective stress friction angle of the material.

Experimental and predicted values of S_u/σ'_{v0} (n.c.) for isotropically-consolidated soils are compared in Fig. 5. Considering the wide variety of sources, the Cam-Clay prediction appears exceptionally good and on the conservative side. A linear regression analysis conducted between actual and predicted values

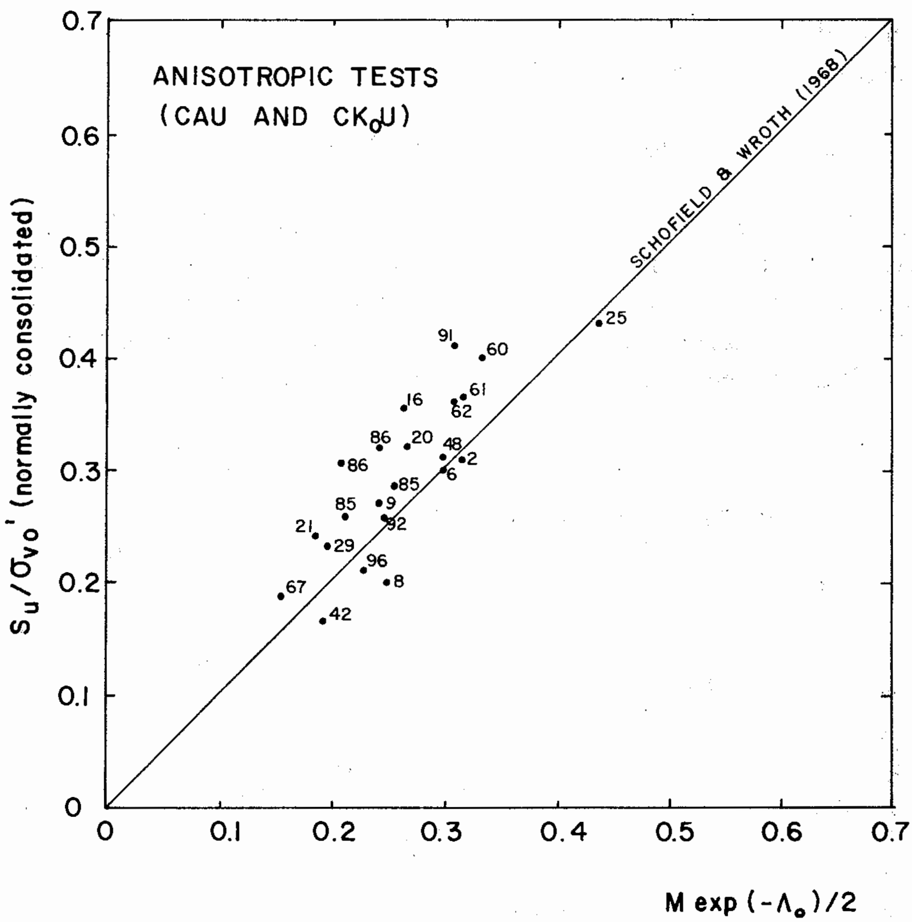

FIG. 6.—Comparison Between Measured and Predicted S_u/σ'_{v0} for Anisotropically Normally-Consolidated Clays

from CIU test results indicated a best fit line (assumed intercept = 0):

$$\frac{S_u}{\sigma'_{v0}} \text{(n.c.) CIU} = 1.123 \frac{S_u}{\sigma'_{v0}} \text{(n.c.) predicted} \quad \ldots \quad (4)$$

with a sample correlation coefficient $r = 0.820$. (Note that $r = 1$ indicates a perfect fit; $r = 0$ indicates no correlation.)

As a first approximation, the same theory may be used to estimate S_u/σ'_{v0} (n.c.) for anisotropically-consolidated clays (refer to Fig. 6). A best fit line ($r = 0.845$) determined from the CAU and CKoU data indicated:

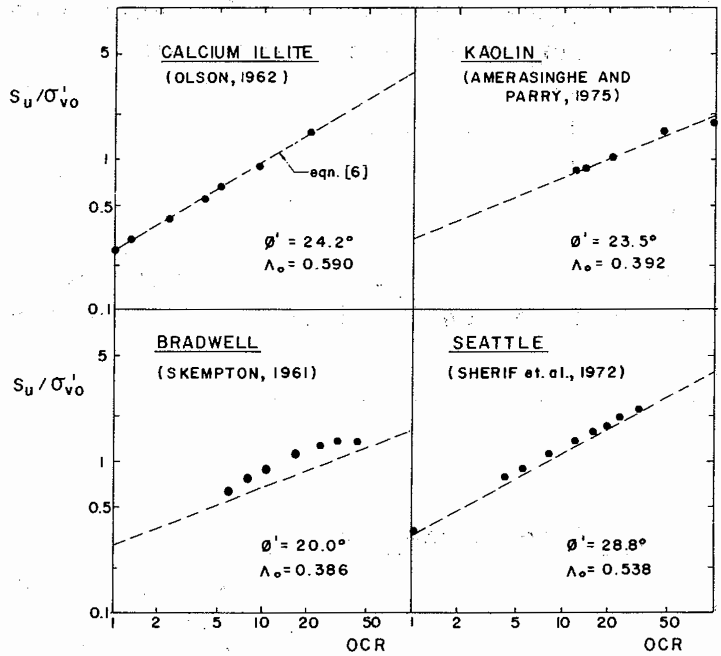

FIG. 7.—Variation of Undrained Strength with OCR for Calcium Illite (55), Kaolin (5), Bradwell Clay (73), and Seattle Clay (67); Experimental (points) and Theoretical (dashed lines)

$$\frac{S_u}{\sigma'_{v0}}\text{(n.c.) CAU} = 1.076\,\frac{S_u}{\sigma'_{v0}}\text{(n.c.) predicted} \quad (5)$$

Eqs. 1 and 3 can be combined to represent S_u/σ'_{v0} for any OCR in terms of effective stresses:

$$\frac{S_u}{\sigma'_{v0}} = \frac{(3\sin\phi')}{(3-\sin\phi')}(e^{-1}\,\text{OCR})^{\Lambda_0} \quad (6)$$

Eq. 6 has been used to predict the effect of OCR on the undrained strength of four different soils (two natural and two remolded) as shown in Fig. 7.

Critical-State Pore Pressure Parameter

The difference between Λ_0 experimentally determined from S_u/σ'_{v0} data and the approximate theoretical value of $\Lambda_0 = 1 - C_s/C_c$ determined from standard consolidation tests is indicated in Fig. 8. It is postulated by the writer (47) that this discrepancy occurs because of problems in properly determining the swelling index parameter (C_s). During most routine laboratory testing, little attention is given to defining a value or range of values for C_s. It appears that the swelling index is actually nonlinear on a plot of void ratio and log-effective stress as noted by Pender (58) and Mesri, Ullrich, and Choi (50). Moreover, the Cam-Clay theory was developed for isotropically-consolidated soils and, in general, C_s has been determined from the results of one-dimensional (anisotropic) consolidation. Therefore, it appears more prudent to determine Λ_0 from the results of shear tests than from consolidation tests.

For consolidated-undrained triaxial shear tests with pore pressure measurements, the value of Λ_0 may be determined for normally-consolidated and overconsolidated soils using an effective stress approach:

$$\Lambda_0 = \frac{\ln\left[\left(\frac{2}{M}\right)\left(\frac{S_u}{\sigma'_{v0}}\right)\right]}{\ln[\mathrm{OCR}] - 1} \quad (7)$$

in which the parameter $M = (6 \sin \phi')/(3 - \sin \phi')$. Therefore, if effective stress methods are used, only one test is required to determine the necessary

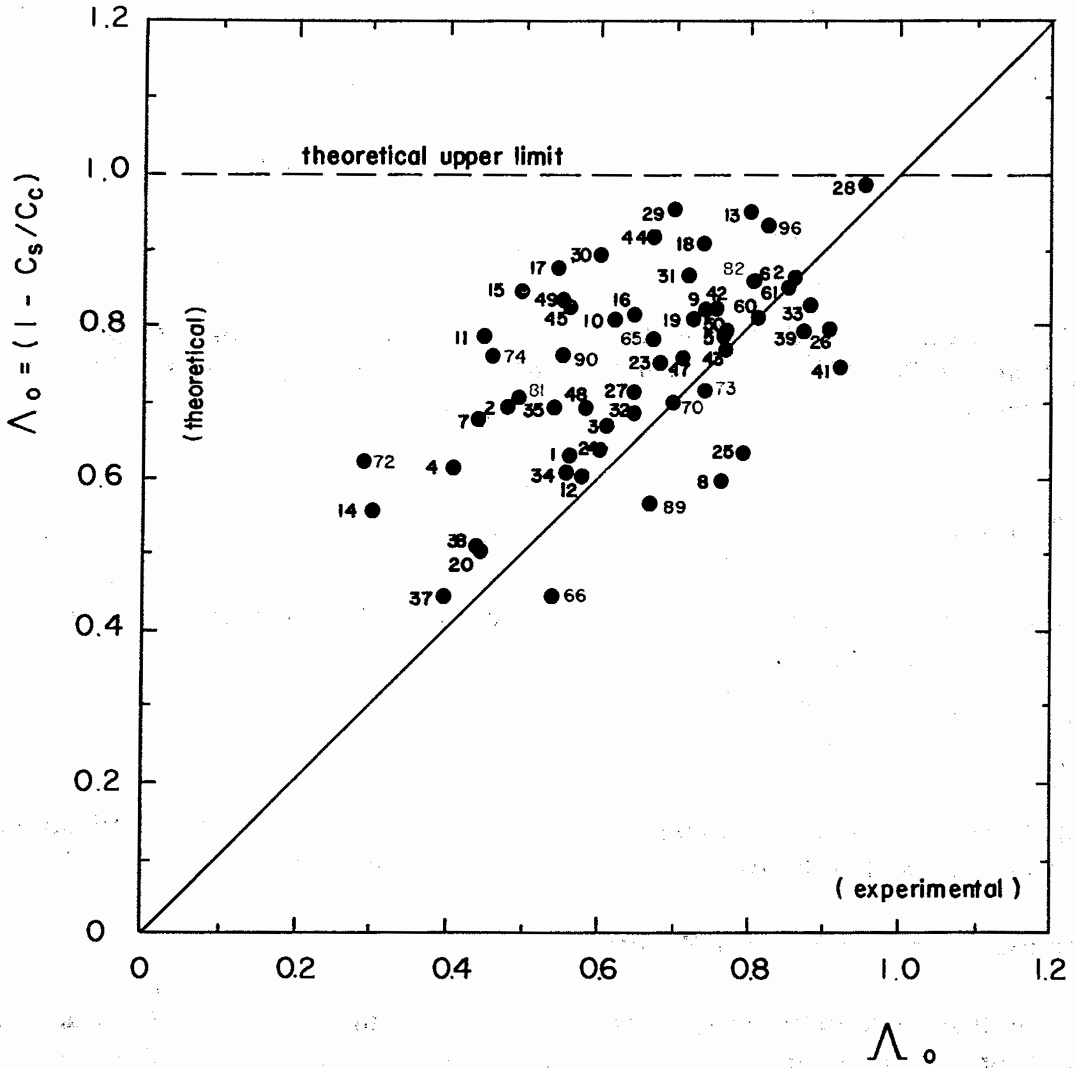

FIG. 8.—Comparison Between Λ_0 Determined From Consolidation Tests and From Undrained Shear Tests

parameters that is: ϕ' and Λ_0. In order to account for the observed experimental-theoretical differences of the Cam-Clay model, it is recommended that an "attraction" of approx 10% be added to the value of the effective vertical confining stress (σ'_{v0}).

Often, natural soils are lightly to heavily overconsolidated in situ. Undrained shear tests are typically conducted at effective confining stress levels approximately equal to the effective overburden stresses, or alternatively, using a

SHANSEP approach. Since the OCR may not be known for a natural soil, the pore pressure parameter may be found from the results of two or more shear tests conducted at different initial vertical confining stress levels, assuming the samples have the same preconsolidation pressure ($\sigma'_{v_{max}}$) in common. Thus

$$\Lambda_0 = \frac{\log\left[\dfrac{S_u}{\sigma'_{v02}}\right] - \log\left[\dfrac{S_u}{\sigma'_{v01}}\right]}{\log\left[\dfrac{\sigma'_{v01}}{\sigma'_{v02}}\right]} \qquad (8)$$

in which $\sigma'_{v02} < \sigma'_{v01} < \sigma'_{v_{max}}$. In other words, the parameter Λ_0 is defined by the slope of a linear relationship between $\log\ [S_u/\sigma'_{v0}]$ and $\log\ [1/\sigma'_{v0}]$ as

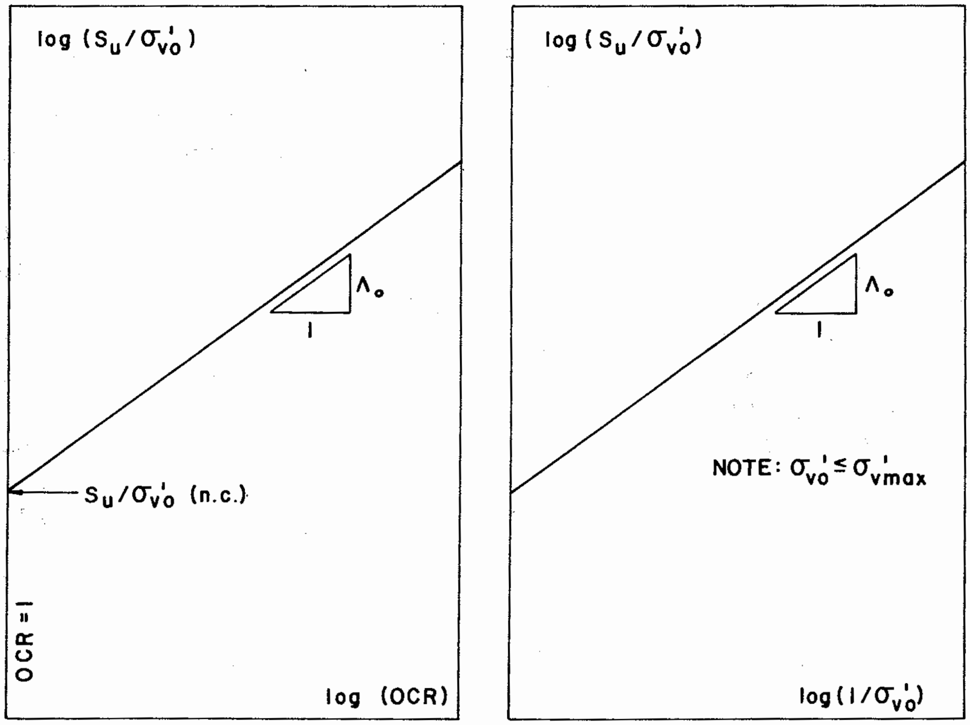

FIG. 9.—Definitions of the Critical-State Pore Pressure Parameter for Both Known and Unknown OCR

shown in Fig. 9. The in situ OCR can then be backcalculated for these confining stress levels from:

$$\mathrm{OCR} = \left[\left(\frac{2}{M}\right)\frac{S_u}{\sigma'_{v0}}\,(\mathrm{o.c.})\,\exp\,(\Lambda_0)\right]^{(1/\Lambda_0)} \qquad (9)$$

Since a minimal number of tests are needed to define the characteristics of the soil, the laboratory testing program for a geotechnical project can be used more effectively to investigate the variability of the natural soil deposits at a given site.

Statistical Trends

Since a large amount of data was compiled for this study, an extensive series of linear regression analyses were conducted between the various soil properties in Table 1 in hopes of discerning any correlative trends. Statistical relationships that were determined to be significant in this study are summarized in Table 2. Three soils were not included in certain of the regression studies due to their high compressibility (No. 28) and high plasticity characteristics (Nos. 14 and 25). A relatively good relationship (r = 0.722) appears to exist between S_u/σ'_{vo} (n.c.) and sin ϕ' for the data. Faint trends (r = 0.3–0.4) were observed between the parameter Λ_0 and index properties of the soils. However, a more significant relationship (r = 0.650) was found to exist between the product $C_c \Lambda_0$ (theoretically equal to $C_{si} - C_{ci}$) and index properties. In addition, sin ϕ' appeared to correlate (r = 0.583) with the ratio of plasticity index to liquid limit (PI/LL).

TABLE 2.—Results of Linear Regression Analyses

Statistical trend (1)	Number of data points (2)	Sample correlation coefficient (3)
Su/σ'_{vo} (n.c.) all = 0.923 $(0.5Me^{-\Lambda_0})$ + 0.059	98	0.842
Su/σ'_{vo} (n.c.) CIU = 0.909 $(0.5Me^{-\Lambda_0})$ + 0.067	80	0.839
Su/σ'_{vo} (n.c.) CAU = 0.952 $(0.5Me^{-\Lambda_0})$ + 0.036	18	0.868
$\Lambda_0 = 0.805\ (1 - Cs/Cc) + 0.035$	59	0.605
$C_c = (\mathrm{PI} + 26)/138$	56	0.745
$C_c = (\mathrm{LL} - 13)/109$	56	0.813
$C_c\Lambda_0 = (\mathrm{PI} + 12)/172$	58	0.648
$C_c\Lambda_0 = (\mathrm{LL} - 4)/213$	58	0.662
Su/σ'_{vo} (n.c.) all = 0.642 sinϕ' + 0.031	98	0.722
sin ϕ' = 0.656 − 0.409 (PI/LL)	87	0.538

Summary

For isotropically-consolidated remolded clays, Λ_0 appears to be totally independent of OCR. Yudhbir and Varadarajan (92) have confirmed that Eq. 6 holds true for values of OCR ranging from 1–375 for Kanpur Clay (see Fig. 10). For natural soils that are subjected to anisotropic consolidation, however, a limiting value of OCR would be expected. During anisotropic swelling, K_0 increases with increasing OCR until K_0 equals K_p (the coefficient of passive earth pressure). This is evidenced by the Bradwell London Clay reported by Skempton (73) and shown in Fig. 7. Bradwell Clay shows a decrease in undrained strength after apparently experiencing passive failure at an OCR of about 30.

The advantage of using the critical-state pore pressure parameter (Λ_0) over Skempton's pore pressure parameter (A_f) and Henkel's pore pressure (α) is that Λ_0 is constant over a range of OCR values while A_f and α are variable

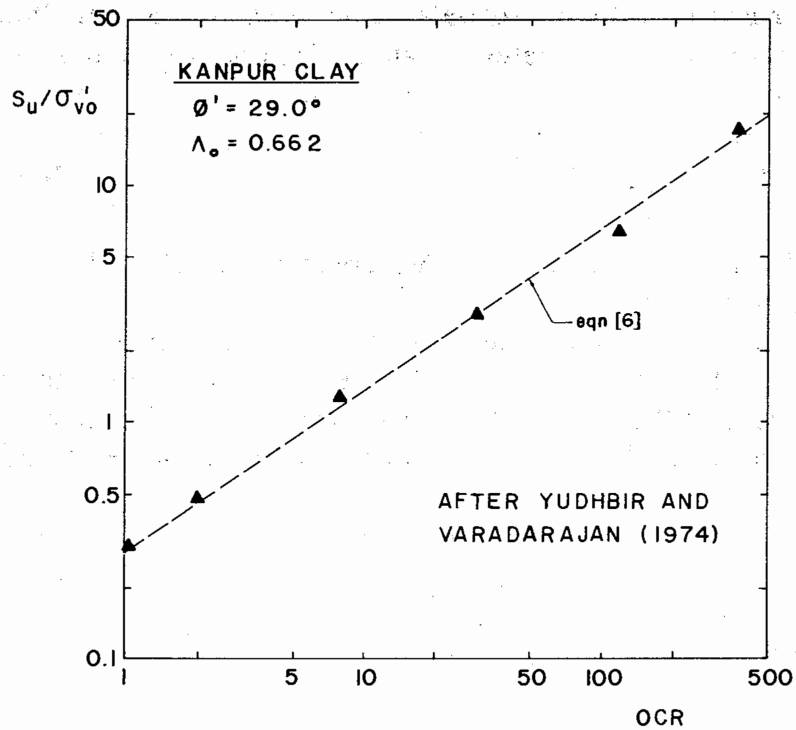

FIG. 10.—Experimental and Theoretical Variation of S_u/σ'_{v0} for OCR $\leq$ 375 on Isotropically-Consolidated Kanpur Clay (92)

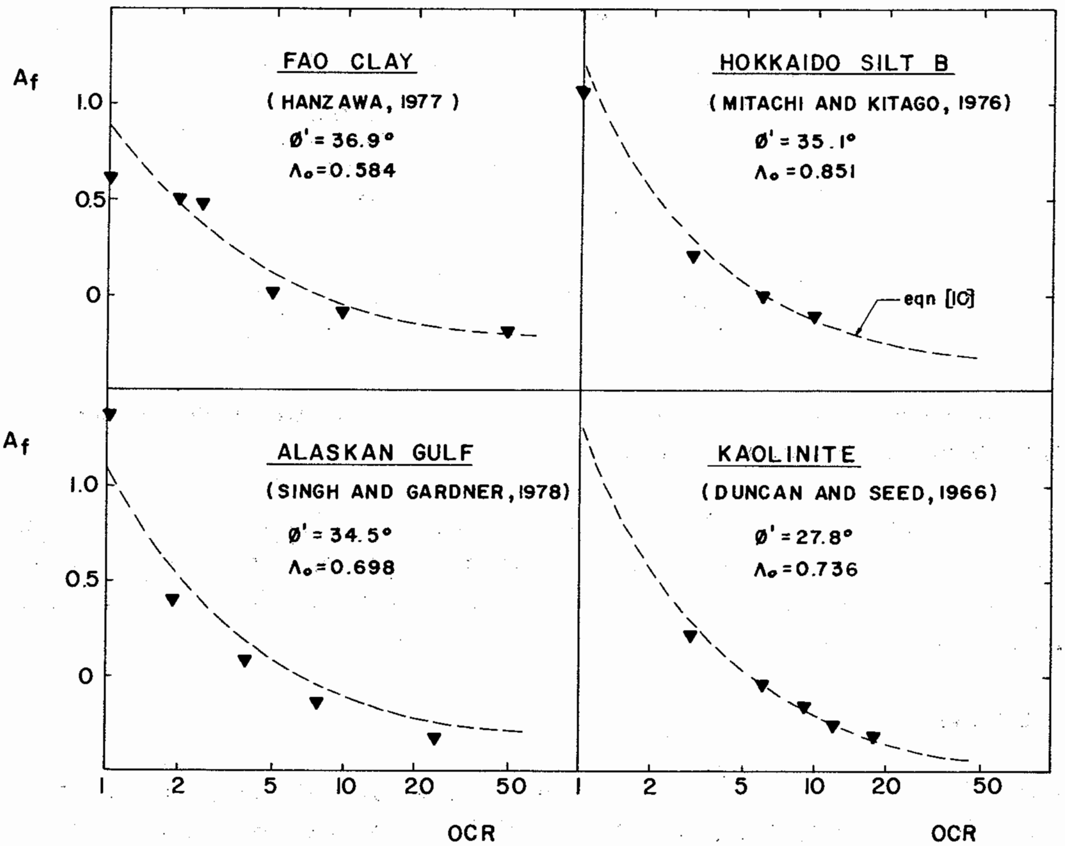

FIG. 11.—Effect of OCR on Skempton's A Parameter at Failure for Fao Clay (27), Hokkaido Silt (51), Alaskan Gulf (72), and Kaolinite (20); Experimental (points) and Theoretical (dashed lines)

with OCR. For isotropically-consolidated soils sheared under undrained conditions, the pore pressure parameter at failure (A_f) as a function of OCR may be expressed by:

$$A_f = \left[\left(\frac{e}{\mathrm{OCR}} \right)^{\Lambda_0} - 1 \right] M^{-1} + \frac{1}{3} \quad \text{(10)}$$

Measured and predicted values of A_f are compared in Fig. 11 for two natural and two laboratory prepared soils.

The commonly accepted parameter (A) is also known to vary with stress level to failure and initial stress state (K_0). Pender (58), VanEekelen and Potts (84), and Mayne and Swanson (48) have shown Λ_0 to be independent of stress level. It has also been shown that Λ_0 is essentially the same for isotropic and

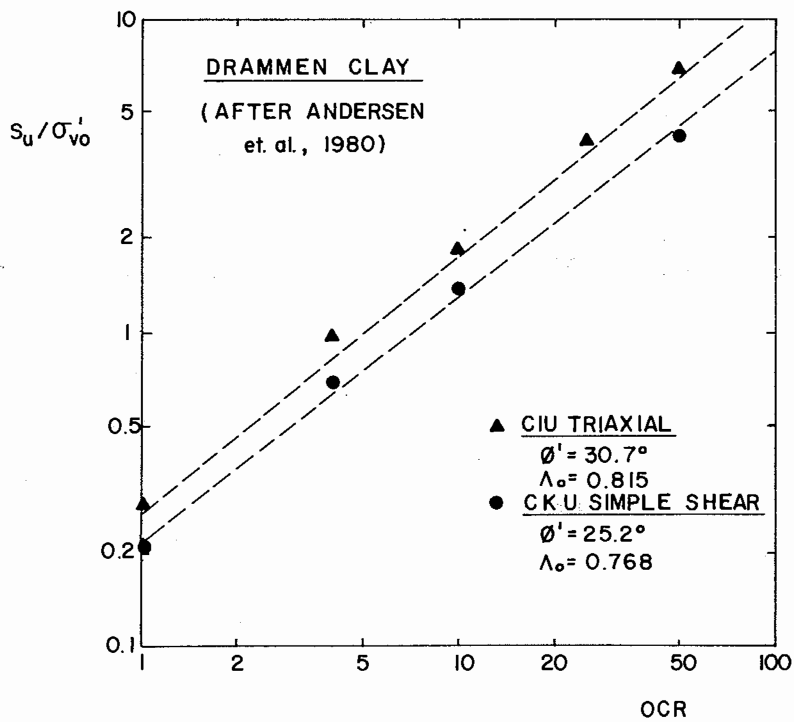

FIG. 12.—Measured and Predicted S_u/σ'_{v0} with OCR for Isotropically and Anisotropically Consolidated Drammen Clay (6)

anisotropic tests (48,51). In Fig. 12, data on Drammen Clay reported by Anderson et al. (6) show little variation in Λ_0 determined from CIU and CK_oU tests, even though two different test apparatus were used including: standard triaxial and direct simple shear.

Conclusions

The critical-state Cam-Clay theory gives rational and reasonable predictions of undrained strength for normally-consolidated to overconsolidated clay soils. The method is simple to apply, requiring only two soil constants (ϕ' and Λ_0) in order to predict the effect of OCR on undrained shear strength to overburden ratio (S_u/σ'_{v0}). If the OCR is known and pore pressure measurements are made, these constants may be obtained from the results of only one consolidated-undrained shear test. A minimum of two tests is required if the stress history (OCR) is not known. Although originally developed for isotropically-consolidated soils, it is shown that the method can be used to give estimates of undrained strength for anisotropically-consolidated clays and silts.

The validity of the theory is shown to be well supported by data collected from many different sources throughout the world. In addition, it is shown that the method is an effective stress approach that encompasses total stress analyses such as SHANSEP. Therefore, it is believed that the Cam-Clay theory has wide applicability in geotechnical modelling of natural and artificial clay soil deposits.

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Appendix II.—Notation

The following symbols are used in this paper:

A_f = Skempton's pore pressure parameter at failure;
C_c = one-dimensional virgin compression index;
C_{ci} = isotropic virgin compression index;
C_s = one-dimensional swelling or rebound index;
C_{si} = isotropic swelling index;
K_0 = coefficient of earth pressure at rest;
K_p = coefficient of passive earth pressure;

M = $(6 \sin \phi')/(3 - \sin \phi')$ = slope of failure line in Cambridge q-P space;
OCR = overconsolidation ratio (in terms of effective vertical stresses);
P' = mean normal effective stress;
q = deviator stress;
r = sample correlation coefficient;
S_u/σ'_{v0} = normalized undrained strength to overburden ratio;
α = Henkel's pore pressure parameter;
ϕ' = effective stress friction angle;
Λ_0 = critical-state pore pressure parameter;
σ'_{v0} = overburden pressure or initial effective vertical confining stress; and
$\sigma'_{v_{max}}$ = maximum vertical preconsolidation pressure.

15816 CAM-CLAY PREDICTIONS OF UNDRAINED STRENGTH

KEY WORDS: **Clays;** Laboratory tests; **Normally consolidated soils; Overconsolidated soils;** Pore water pressure; **Shear strength; Shear tests;** Soils; Static loads; **Stresses; Total stress;** Triaxial tests

ABSTRACT: The validity of the critical-state Cam-Clay theory to predict undrained shear strength is substantiated with data from 96 different clay and silt soils reported in the geotechnical literature. The study investigates normally-consolidated and overconsolidated strengths, which is important in that a range of strengths is more common in natural soil deposits. Attention is given to establishing the critical-state pore pressure parameter from routine consolidated-undrained shear tests. The critical-state theory is shown to be an effective stress method that incorporates total stress approaches similar to SHANSEP.

REFERENCE: Mayne, Paul W., "Cam-Clay Predications of Undrained Strength," *Journal of the Geotechnical Engineering Division,* ASCE, Vol. 106, No. GT11, **Proc. Paper 15816**, November, 1980, pp. 1219-1242

Errata.—The following corrections should be made to the original paper:

Page 1103, line 8 from the bottom: Should read "transducers" instead of "diaphragm transducers"
Page 1106, line 1: Should read "Normally consolidated specimens" instead of "Normally, consolidated specimens"
Page 1119, lines 8–9: Should read "Marine Soils under Cyclic Loading" instead of "Two Marine Clays during Cyclic Loading"

Cam-Clay Predictions of Undrained Strength[a]

Discussion by Thomas C. Anderson,[2] M. ASCE

The writer agrees with the author as to the importance of being able to predict the undrained strength of overconsolidated and normally-consolidated clays, especially in light of the problems associated with sampling, testing, and costs. However, the writer wishes to discuss the validity of the critical-state Cam-Clay concept in predicting the in-situ strength ratio.

For normally-consolidated clays, Skempton (96) established the following empirical correlation based on field vane tests

$$\frac{S_u}{\sigma_{vo'}} \text{ (normally-consolidated)} = 0.11 + 0.0037\,(PI) \quad \ldots \quad (11)$$

Bjerrum's study (93) of embankments on soft ground foundations that failed under undrained conditions indicated that a reduction factor, C_r, should be applied to the results of undrained laboratory tests or field vane tests on clays of high plasticity. The reduction factor may be approximated by the following equation (95)

$$C_r = 1.0 - 0.5 \log\left(\frac{PI}{20}\right), \quad \text{for} \quad PI \geqq 20 \quad \ldots \quad (12)$$

Thus, the in-situ undrained strength ratio for normally-consolidated clays can be estimated from Eq. 11, appropriately corrected by Eq. 12. Fig. 13 is a plot of $S_u/\sigma_{vo'}$ (n.c.) versus PI for all of the data presented in Table 1. Skempton's correlation (Eq. 11) has been plotted on Fig. 13, along with the corrected version by Eq. 12. The data points in Fig. 13 show no apparent relationship with plasticity index, which is contrary to that indicated by Gardner (94). In addition, 76 and 86% of the data points, respectively, lie above the uncorrected and corrected field strength ratio curves. Therefore, it appears that the Cam-Clay model generally

[a] November, 1980, by Paul W. Mayne (Proc. Paper 15816).
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overpredicts the field undrained strength ratio for normally-consolidated clays. The reason for this probably is due to the fact that nearly all of the data in Table 1 was developed from consolidated undrained triaxial compression (CIU or CK_oU) tests. As pointed out by Ladd, et al. (41), "CU triaxial compression tests significantly overestimate the average in-situ S_u for stability analyses with most clays."

With regard to the undrained strength of overconsolidated clays, Eq. 1 is the basic relationship employed by the author. It sould be noted that this equation is remarkably similar to the following expression presented by Ladd, et al. (41).

$$\frac{\dfrac{S_u}{\sigma_{vo'}} \text{ (overconsolidated)}}{\dfrac{S_u}{\sigma_{vo'}} \text{ (normally-consolidated)}} = \text{OCR}^m \quad \ldots \quad (13)$$

in which $m \cong 0.8$. The data presented in Table 1 and discussed by the author

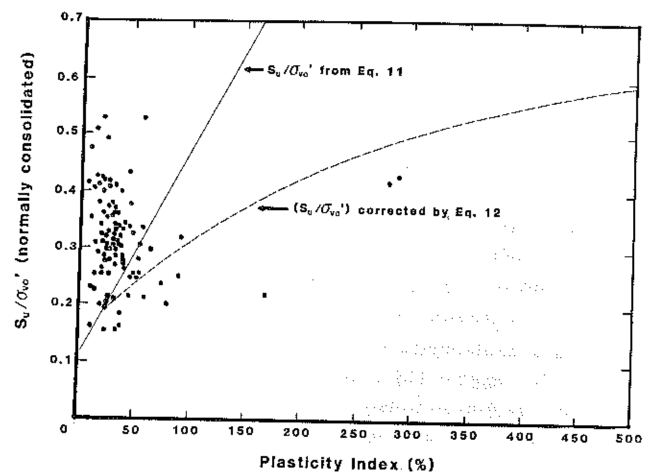

FIG. 13.—S_u/σ_{vo} Versus Plasticity Index for Normally-Consolidated Clays

indicate the exponent Λ_o in Eq. 1 varies from 0.130 to 0.998 with a mean of 0.64. Thus, even though Eqs. 1 and 13 are basically the same, the OCR exponent values differ significantly.

Lastly, please note that Eq. 13 can be utilized in the following manner as a very simple prediction method for the undrained strength ratio of overconsolidated clays:

$$\frac{S_u}{\sigma_{vo'}} \text{ (overconsolidated)} = \frac{S_u}{\sigma_{vo'}} \text{ (normally-consolidated)} \times \text{OCR}^m \quad \ldots \quad (14)$$

in which $S_u/\sigma_{vo'}$ (normally-consolidated) = Skempton's corrected correlation.

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Discussion by Demetrious C. Koutsoftas,[3] A. M. ASCE

The author made a serious effort to collect and analyze a massive amount of laboratory test results. This information will prove to be a useful reference for practitioners. The data were obtained from many sources and apparently different investigators used different types of tests. This is not clearly pointed out in the paper and furthermore it is not adequately accounted for in the analysis of the data. The discusser will attempt to demonstrate that the type of test has an important effect on measured strength parameters and has to be properly accounted for in the analysis of the data; this is perhaps most important when one attempts to apply Eq. 3 to estimate normalized strength ratios, $Su/\bar{\sigma}_{vo}$, as proposed by the author.

As far as the discusser could ascertain, Table 1 includes data from isotropically consolidated triaxial compression tests ($C\overline{IU}C$), anisotropically consolidated triaxial compression ($C\overline{AU}C$), K_0-consoliated triaxial compression ($CK_o\overline{U}C$), and K_0-consolidated direct simple shear tests ($CK_o\overline{UD}SS$). Although there is evidence to suggest (19,100) that $Su/\bar{\sigma}_{vo}$ ratios measured in triaxial compression tests are not seriously influenced by the mode of consolidation prior to shear, there is ample evidence (101,41,95,98) to demonstrate that for many clays, strength ratios from $CK_o\overline{UD}SS$ tests are normally lower than corresponding values measured from triaxial compression tests. Data presented by Ladd (101) show that $Su/\bar{\sigma}_{vo}$ from compression tests may be 1.1 to 1.7 times the corresponding values obtained from direct simple shear tests. Therefore, it is important to identify in Table 1 which soils were tested in compression ($CK_o\overline{U}C$) and which soils were tested in direct simple shear ($CK_o\overline{UD}SS$).

To the discusser's knowledge, the data reported in Table 1 for soils No. 6, 8, 9, 21, 42, 71, 82, and 96 were obtained from $CK_o\overline{UD}SS$ tests. It should be noted that for soil No. 96, $CK_o\overline{UD}SS$ tests gave lower values of $Su/\bar{\sigma}_{vo}$ than CIU tests. This is further substantiation that the type of test has an important effect on normalized strength ratios.

The results of direct simple shear tests are difficult to interpret in terms of effective stresses (102,103), particularly the determination of friction angle, $\bar{\phi}$. It is therefore important to clarify whether the friction angles reported in Table 1 for the $CK_o\overline{UD}SS$ tests were obtained from interpretation of the $CK_o\overline{UD}SS$ tests or from triaxial tests.

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The data presented in Table 1 require clarification with respect to the following:

1. Soils No. 2 and 18 appear to have the same index properties and the reported friction angles are approximately the same (within experimental error). The tests were performed by the same group of researchers and therefore one wonders if the same soil was used in both cases. If so, the large difference in reported $Su/\bar{\sigma}_{vo}$ values is worthy of comment.
2. The data for soil No. 71 are reported as obtained from CIU tests. Ref. 22, which is reported by the author as the source of this information, does not contain any CIU test data. It would appear that this soil is the same as soil No. 96 and that the tests were $CK_o\overline{UD}SS$. It also appears that the CIU tests reported for soil No. 50 are from the same source as for soil No. 96, which were originally reported in Ref. 93. Yet, slightly different values of $Su/\bar{\sigma}_{vo}$ and Λ_o are reported.
3. The data reported in Table 1 for soil No. 29, Portsmouth clay, indicate a value of $Su/\bar{\sigma}_{vo}$ of 0.23, whereas the data reported in Ref. 69 (the source of this information) indicate a value of $Su/\bar{\sigma}_{vo}$ obtained from compression tests of 0.35; Ladd et al. (41) reported $Su/\bar{\sigma}_{vo}$ values of 0.26 obtained from $CK_o\overline{UD}SS$ tests.
4. Leathers and Ladd (44) reported $Su/\bar{\sigma}_{vo}$ values for soil No. 57 of 0.28 and 0.17, obtained from plane strain compression tests and $CK_o\overline{UD}SS$ tests, respectively. The value of 0.328 reported by the author was apparently obtained from FV tests. The use of FV data would create problems in analyzing these data on the basis of Eq. 3. The author's comments as to how the data were interpreted will be appreciated.
5. Saxena et al. (65) reported undrained strength ratios of 0.158 (average of two tests) from $CK_o\overline{UD}SS$ tests, 0.235 to 0.25 from triaxial compression and 0.14 to 0.16 from triaxial extension; normalized strength ratios of 0.28 were reported from plane strain compression and 0.25 to 0.26 from plane strain extension. It is quite obvious that the type of test has an important effect on the value of the normalized strength ratio.

The value of effective friction angle of 19° reported in Table 1 was apparently determined from $CK_o\overline{U}C$ tests, at the maximum shear stress. However, at the maximum principal stress ratio $(\bar{\sigma}_1/\bar{\sigma}_3)_{max}$, the friction angle reported by Saxena et al. (65) was 30°. Furthermore, the effective stress paths indicate that the maximum shear stress must have occurred at lower strains than the maximum principal stress ratio. It seems that under these conditions, the friction angle determined at the maximum principal stress ratio should be used for critical state analysis. The use of a friction angle of 19° requires justification.

A comprehensive series of undrained shear tests were performed under the discusser's direction, including triaxial compression, triaxial extension, and direct simple shear tests on K_0-consolidated specimens of two marine clays. Pertinent data are presented in Table 3, including $Su/\bar{\sigma}_{vo}$ values computed on the basis of Eq. 3, and the ratio of computed to measured shear strength ratios. The computed $Su/\bar{\sigma}_{vo}$ ratios are 0.80 to 1.57 times the measured values. The biggest

TABLE 3.—Measured and Predicted Strength Ratios for Two Holocene Clays

Soil type (1)	Liquid limit, as a percentage (2)	PI (3)	Type of test (4)	Measured Properties $(Su/\bar{\sigma}_{vo})m$ (5)	Measured Properties $\bar{\phi}$ (6)	Measured Properties Λ_o (7)	Computed $(Su/\bar{\sigma}_{vo})c$ (8)	$[(Su/\bar{\sigma}_{vo})c]/[(Su/\bar{\sigma}_{vo})m]$ (9)
Plastic Holocene	65 ± 10	40	$CK_o\bar{U}C$	0.322	35°	0.78	0.325	1.01
			CK_o UDSS	0.250	30[a]	0.80	0.270	1.08
			$CK_o\bar{U}E$	0.20	31.0	0.88	0.257	1.29
Silty Holocene	35 ± 10	18 ± 5	$CK_o\bar{U}C$	0.325	29.2	0.80	0.26	0.80
			CK_o UDSS	0.230	30[a]	0.88	0.25	1.09
			$CK_o\bar{U}E$	0.175	31.7	0.84	0.28	1.57

[a] Assumed for purposes of computation.

discrepancy appears to be for extension tests.

Based on data shown in Table 3 and data presented on Figs. 5 and 6, it is concluded that the proposed Eq. 3 may seriously overestimate normalized strength ratios for extension tests, and underestimate corresponding values for compression tests.

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Discussion by J. A. Ramalho-Ortigão,[4] M. ASCE and Leandro Moura Costa-Filho[5]

The author has presented an interesting application of the Cam-Clay theory for the prediction of undrained strength and is to be complimented for the huge task of collecting data from many different soils for his analysis. He included geotechnical data on Rio de Janeiro soft gray clay deposit near Guanabara Bay (soil No. 88) presented by Costa-Filho, Werneck and Collet (12). Since then, additional information has been collected for the geotechnical characteristics of this soft clay deposit at the same site by Collet (106) and Ramalho-Ortigão (107) and some of this data is presented here.

A geotechnical profile of the site is shown in Fig. 14. It can be seen that the clay deposit consists of an upper crust about 2 m thick followed by softer

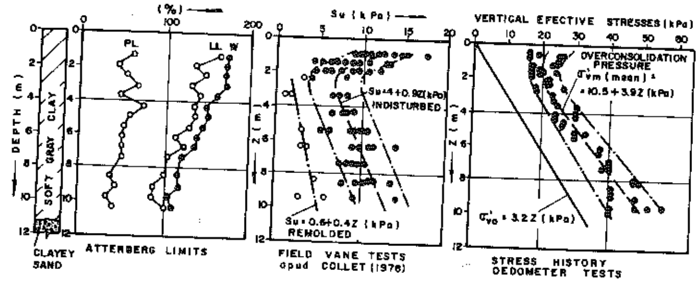

FIG. 14.—Summary of Geotechnical Properties of Rio de Janeiro Soft Gray Clay

material. Values of overconsolidation pressure obtained from oedometer tests, also shown in Fig. 14, indicate that the deposit is lightly overconsolidated. Costa-Filho et al. (12) carried out triaxial CIU compression tests and UU tests. The CIU tests were carried out only on samples consolidated to stresses well above the in situ vertical effective stress, i.e., in the normally consolidated state. From the information presented by the same authors for the UU tests, it would be difficult to obtain a relationship between normalized undrained strength and OCR. This is due to difficulties in defining the maximum past effective stress which varies with depth, as indicated in Fig. 14. It follows, therefore, that the pore pressure parameter, Λ_o, only can be obtained indirectly from this data. Taking $(S_u/\sigma'_{vo})_{NC} = 0.253$ and $\phi' = 25.1°$, as given by Costa-Filho et al. (12), and using Eq. 3 or 7, one gets $\Lambda_o = 0.66$, which is different from the value $\Lambda_o = 0.88$ quoted by the author for the Rio de Janeiro soft clay.

From the results of a large number of odometer tests presented by Ramalho-Ortigão (107), the following mean values can be obtained:

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[5] Asst. Prof., Dept. of Civ. Engrg., Catholic Univ. of Rio de Janeiro, Brasil.

$$\frac{C_c}{1 + e_o} = 0.40 \text{ (at the beginning of the virgin curve)}; \quad \frac{C_s}{1 + e_o} = 0.06 \quad \text{. . . . (15)}$$

Thus, $C_s/C_c = 0.15$, and one gets $\Lambda_o = 0.85$ from the expression $\Lambda_o = 1 - C_s/C_c$. Using the equation statistically obtained by the author

$$\Lambda_o = 0.805\left(1 - \frac{C_s}{C_c}\right) + 0.035 \quad \text{. . . . . (16)}$$

a value of 0.72 is obtained.

In Fig. 15, a comparison is made between the values of the slopes of the OCR versus the normalized strength curve in a log-log plot. This figure includes the results of CIU and CK_oU triaxial tests carried out by Ramalho-Ortigão

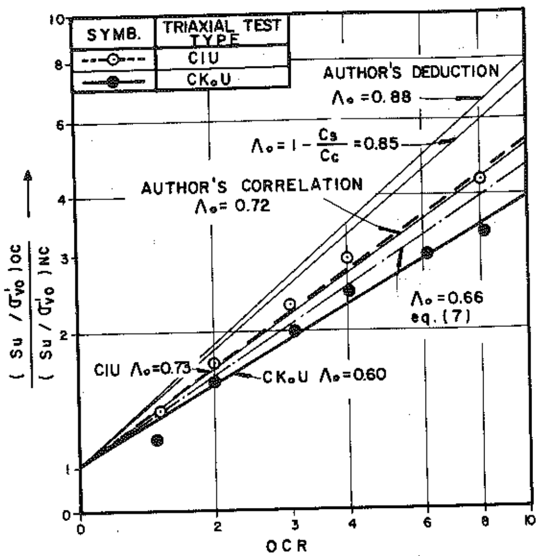

FIG. 15.—Relationship Between Normalized Undrained Strength and OCR for Rio de Janeiro Soft Gray Clay

(107), and the previously estimated values of Λ_o. The laboratory results follow approximately straight lines with slopes of 0.73 and 0.60 for the CIU and CK_oU tests, respectively. These values are in good agreement with the theoretical one ($\Lambda_o = 0.66$) using Eqs. 3 or 7, and with the value obtained from the statistical correlation, $\Lambda_o = 0.72$. The difference between those values, and the value $\Lambda_o = 0.85$ obtained from the odometer test, fall within the range of data presented by the author in Fig. 8. However, the value quoted by the author in Table 1 of the paper does not fall in the range of the laboratory results of the normalized undrained strength versus OCR for the Rio de Janeiro gray clay.

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The original and three copies of the Discussion should be submitted on 8-1/2-in. (220-mm) by 11-in. (280-mm) white bond paper, typed double-spaced with wide margins. The length of a Discussion is restricted to two *Journal* pages (about four typewritten double-spaced pages of manuscript including figures and tables); the editors will delete matter extraneous to the subject under discussion. If a Discussion is over two pages long it will be returned for shortening. All Discussions will be reviewed by the editors and the Division's or Council's Publications Committees. In some cases, Discussions will be returned to discussers for rewriting, or they may be encouraged to submit a paper or technical note rather than a Discussion.

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Discussions have a specific format. The title of the original paper/technical note appears at the top of the first page with a superscript that corresponds to a footnote indicating the month, year, author(s), and number of the original paper/technical note. The discusser's full name should be indicated below the title (see Discussions herein as an example) together with his ASCE membership grade (if applicable).

The discusser's title, company affiliation, and business address should appear on the first page of the manuscript, along with the *Proceedings* paper number of the original paper/technical note, the date and name of the *Journal* in which it appeared, and the original author's name.

Note that the discusser's identification footnote should follow consecutively from the original paper/technical note. If the paper/technical note under discussion contained footnote numbers 1 and 2, the first Discussion would begin with footnote 3, and subsequent Discussions would continue in sequence.

Figures supplied by the discusser should be designated by letters, starting with A. This also applies separately to tables and references. In referring to a figure, table, or reference that appeared in the original paper/technical note use the same number used in the original.

It is suggested that potential discussers request a copy of the *ASCE Authors' Guide to the Publications of ASCE* for more detailed information on preparation and submission of manuscripts.

Cam-Clay Predictions of Undrained Strength[a]

Closure by Paul W. Mayne,[6] A. M. ASCE

Koutsoftas and Anderson are correct in noting that the data are primarily from triaxial compression tests. This is briefly mentioned in the introduction and should have been reiterated throughout the paper.

It is unfortunate that $S_u/\sigma_{vo'}$(nc) values from isotropically (CIUC) and anisotropically (CK_oUC) consolidated undrained triaxial compression tests do not correlate with PI. Anderson is referred to Schmertmann and Morgenstern (116) who have provided good arguments against the use of global empirical correlations with PI. Larsson (112) has arrived at similar conclusions for triaxial compression tests in Scandinavian clays. He recently reviewed reported failures of embankments and foundations, and cautioned that the field vane should be used only as an index test, substantiated by the results of CK_oUC, DSS, and CK_oUE tests. In a separate study, Trak, et al. (117) indicate that "the strength available at failure under an embankment is nearly independent of the plasticity index." Their Fig. 17 is similar to the one presented by Anderson in his discussion.

All values of Λ_o in Table 1 were determined from a total stress approach described by Fig. 9, except for clay from Guanabara Bay reported by Costa-Filho, et al. (12). Eq. 7 was used, corrected by an "attraction" of 12% as discussed, to obtain $\Lambda_o = 0.78$. The value of 0.88 was incorrectly reported by the writer in Table 1. The writer is grateful to Ramalho-Ortigao and Costa-Filho for the additional data on Rio de Janeiro clay.

Koutsoftas has indicated the importance of stress rotation on the undrained shear strength of clay soils. In a manner presented by Soydemir (104) and Koutsoftas and Fischer (110) the CK_oUC, CK_oUDSS, and CK_oUE tests can be used to represent principal stress rotations approximating $\beta = 0°$, 35°–45°, and 90°, respectively. The original Table 1, indicating which soils were tested under CK_oUDSS conditions, was unfortunately edited by the writer prior to submittal to ASCE.

The available data for soils tested under both triaxial compression and direct simple shear were reviewed for comparison. Silty Holocene clay reported by Koutsoftas and Fischer (34) was designated as soil No. 97. The ratio of S_u/σ'_{vo} (nc) from CK_oUDSS to that from CIUC and CK_oUC tests generally varied between 0.6 and 0.8 for normally consolidated specimens. As presented in Fig. 16, however, an interesting relationship developed, independent of OCR:

$$\frac{S_u}{\sigma'_{vo}}(CK_oUDSS) = 0.7\frac{S_u}{\sigma'_{vo}}(\text{CIUC or } CK_oUC\text{, or both}) \quad \ldots\ldots\ldots\ldots \quad (17)$$

However, Prévost (114) has presented theoretical conversions between

[a]November, 1980, by Paul W. Mayne (Proc. Paper 15816).

[6]Geotechnical Engr., Law Engrg. Testing Co., Wash., D.C.

strengths measured under simple shear and triaxial conditions. Although CK_oUDSS testing is advocated by Ladd and Foott (40,109), it is the writer's experience that few commercial laboratories have simple shear equipment.

Addressing the questions raised by Koutsoftas:

1. As explained in Table 1, S_u/σ'_{vo}(nc) for soil No. 18 was estimated by extrapolation from $12 \leq OCR \leq 212$; thereby the difference with soil No. 2 tested at $1 \leq OCR \leq 2.6$.
2. Data for soil Nos. 71 and 50 (Drammen clay) were derived from simple shear and triaxial tests, respectively. Minor differences with respect to soil No. 96 may be attributed to variations in the actual presentations of the data (6,22,84).
3. The data for Portsmouth clay (soil No. 29) was obtained from simple shear tests reported by Ladd (111).
4. The critical-state pore pressure parameter (Λ_o) for New York varved clay was determined using Eq. 8 (Fig. 9) and field vane test data to show applicability of the method to in situ testing. Eq. 3 was not applied in this case.
5. For many soils, the "critical state" failure is difficult to discern. The effective stress method was applied to peak undrained strength where, in general, ϕ' was determined at maximum deviator stress (114). For soil No. 82 reported by Saxena, et al. (65), apparently $\phi' = 15°$ should have been used.

The theory presented for Cam-Clay in Eqs. 3 and 6 are for consolidated-undrained triaxial compression tests. Ninety-two percent of the data presented is derived from CIUC, CK_oUC, and CAUC tests. The generalized prediction for undrained strength by the critical-state theory is

$$\frac{S_u}{\sigma'_{vo}} = \frac{M}{2}(e^{-1}\,\mathrm{OCR})^{\Lambda_o} \qquad (18)$$

in which $M_{\mathrm{compression}} = 6 \sin \phi'/(3 - \sin \phi')$ and $M_{\mathrm{extension}} = 6 \sin \phi'/(3 + \sin \phi')$ as derived by Schofield and Wroth (66). In Fig. 17, Eq. 18 has been

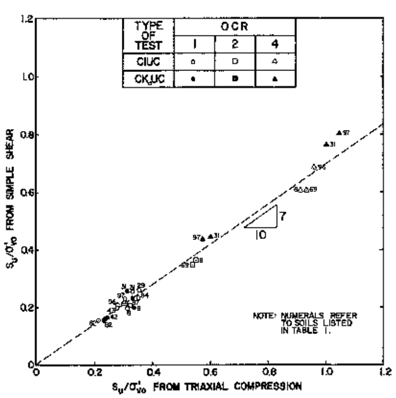

FIG. 16.—Ratio of S_u/σ_{vo} (nc) from CK_oUDSS to that from CIUC and CK_oUC

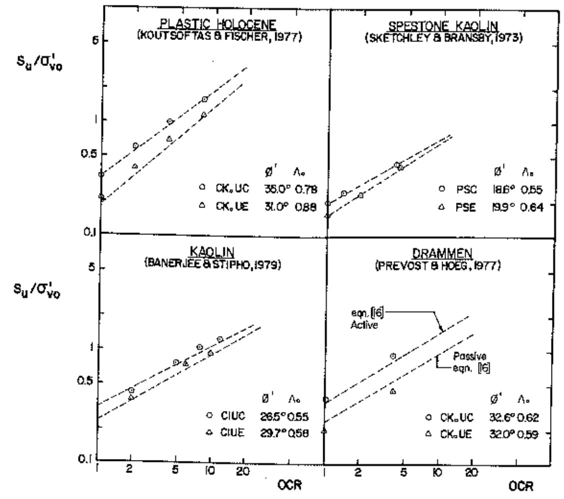

FIG. 17.—$S_u/\sigma_{vo} = M/2(e^{-1}\mathrm{OCR})^{\Lambda_o}$ Applied to Four Soils

applied to four soils, assuming that relatively small differences exist between isotropically-consolidated triaxial, K_o consolidated triaxial, and plane-strain tests. Clearly, the effects of stress rotation can be significant, as predicted by Cam-Clay theory.

The writer, also a practitioner, advocates the use of only two soil constants, ϕ' and Λ_o, for small projects where limited budgets restrict the amount of testing possible. For critical structures, these concepts can be used to organize a large amount of data together or recognize soils which exhibit deviant behavior and justify extensive testing.

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Tension Resistant Inclusions in Soils[a]

Discussion by Robert P. Chapuis[5]

The general analytical approach presented by the authors is an invaluable step in the improvement of design procedures for tensile strain-controlled soil-inclusion systems. The writer wishes to discuss the authors' limited classification of available design methods, and the questions some of the conclusions of the experimental investigations.

According to the way the problem is approached, studies of the mechanical behavior of composite materials made with soil and tension resistant inclusions may be divided into two groups. In the first approach consisting of a decomposition calculus, the forces in the soil and in the inclusions are considered separately. This decomposition calculus was generally restricted to limit-equilibrium analysis, and was used for example by Schlosser and Vidal (13), Schlosser (22) and by Lee, et al. (20) to design reinforced earth retaining walls. In the second approach consisting of a global calculus, the reinforced soil is considered as an anisotropic continuum. This global calculus was used by Westergaard (23) after a suggestion made by Professor Casagrande, and more recently adopted for example by Harrison and Gerrard (5), Romstad, et al. (12). This second approach usually involves fem and is relative to the general mechanical behavior of the reinforced soil. However, the first approach, an analytical one, is the only one which can facilitate the understanding of the mechanical internal behavior and of soil-inclusion interactions. Thus, as done by the authors, it appears useful to develop a fem analytical approach with distinct elements for soil and inclusions.

In the past years, the writer has developed an analytical method (16,17) to take into account the soil-inclusion interactions and, especially, the shear stresses that change their sign above and under a reinforcing element. The influence of these shear stresses is important since they are equivalent to the tensions of inclusions which in turn may be equilibrating an active earth pressure for

[a]December, 1980, by Kamal Z. Andrawes, Alan McGown, Mohsen M. Mashhour and Ragui F. Wilson-Fahmy (Proc. Paper 15928).

[5]Vice-Pres., Mon-Ter-Val Inc., Montréal, Québec, Canada.

example. From these results (16,17) which were restricted to limit-equilibrium, the writer concluded that the replacement (in design) of a reinforced soil by an equivalent continuum should consider the following equivalences: inclusion tensions + soil solicited by inclusions → analytical method. This is equivalent to anisotropic cohesion + soil with cohesion effect → global method. As a matter of fact, and at least for limit-equilibrium, the equivalent continuum of the global method should have an anisotropic cohesion taking into account the concentration and the orientation of the inclusions relative to principal stresses as well as the stress level. Furthermore, it should be able to account for such phenomena as the lack of adherence (the tension developed in an inclusion is lower than its resistance), the cohesion effect (relative to the shear stress distribution around the inclusions), and the loss of adherence (no tension can be developed in the inclusions). Consequently, it is the writer's opinion that

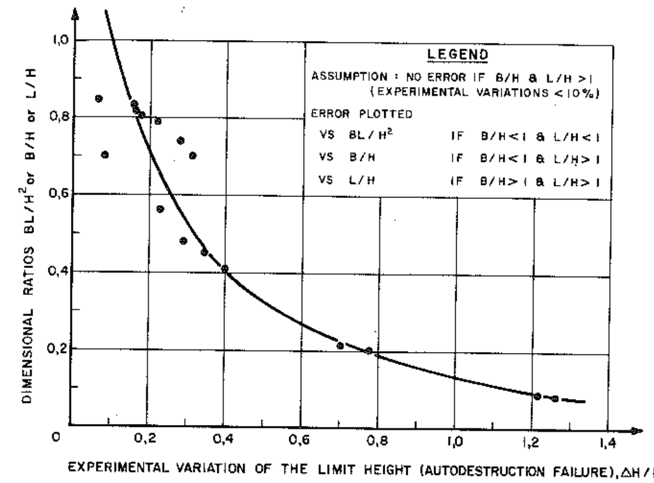

FIG. 14.—Tension Resistant Inclusions in Soils; Influence of the Sand Box Dimensions on the Limit Height, H, of Model Reinforced Earth Retaining Walls $H \approx H_{\ell}$ for $B/J > 1$ and $L/H > 1$

fem can improve design methods only if they use distinct soil and inclusion elements taking into account the stress and strain peculiarities around the inclusions.

The preceding information on design procedures should give a better idea of the work performed by the authors, who in the writer's opinion, should be congratulated for their general analytical approach.

Further comments are relative to the embankment test results, especially in relation to the sand box dimensions and the silo effects. For experiments involving active earth pressure, it is well-known that the internal width must be larger than the height, H, of the fill (15,18). More specifically, tests have been performed in sand boxes similar to that of authors', in the case of reinforced earth retaining walls. These tests (14) have been partially turned to account (19). They have been gathered by the writer to show simultaneously the influences of the width, W, and length, L, of sand box. It appears from Fig. 14 that silo effects are